9.4 Past Institute Exam Questions

1. Apr 00 - Q11
   In any year, the rate of interest on funds invested with a given insurance company is independent of the rates of interest in all previous years. Each year the value of $(1 + i_t)$, where i_t is the rate of interest earned in the tth year, is lognormally distributed. The mean and standard deviation of i_t are 0.07 and 0.20 respectively.

   (a) Determine the parameters μ and σ^2 of the lognormal distribution of $1 + i_t$.

   (b) i. Determine the distribution of S_{15} , where S_{15} denotes the accumulation of one unit of money over 15 years.

       ii. Determine the probability that $S_{15} > 2.5$.

2. Apr 01 - Q9
   The annual yields from a particular fund are independent and identically distributed. Each year, the distribution of $1 + i$ is log-normal with parameters $\mu = 0.07$ and $\sigma^2 = 0.006$, where i denotes the annual yield on the fund.

   (a) Find the mean accumulation in ten years' time of an investment in the fund of £20,000 at the end of each of the next ten years, together with £150,000 invested immediately.

   (b) Find the single amount which should be invested in the fund immediately to give an accumulation of at least £600,000 in ten years' time with probability 0.99.

3. Apr 02 - Q11 A company is adopting a particular investment strategy such that the expected annual effective rate of return from investments is 7% and the standard deviation of annual returns is 9%. Annual returns are independent and $(1 + i_t)$ is lognormally distributed where it is the return in the tth year. The company has received a premium of £1,000 and will pay the policyholder £1,400 after 10 years.

   (a) Calculate the expected value and standard deviation of an investment of 1,000 over 10 years, deriving all formulae that you use.

   (b) Calculate the probability that the accumulation of the investment will be less than 50% of its expected value in ten years' time.

   (c) The company has invested £1,200 to meet its liability in 10 years time. Calculate the probability that it will have insufficient funds to meets its liability.

4. Sep 00 - Q6
   An insurance company calculates the single premium for a contract paying £10,000 in ten years' time as the present value of the benefit payable, at

the expected rate of interest it will earn on its funds. The annual effective rate of interest over the whole of the next ten years will be 7%, 8% or 10% with probabilities 0.3, 0.5 and 0.2 respectively.

  (a) Calculate the single premium.

  (b) Calculate the expected profit at the end of the term of the contract.

5. Sep 00 - Q8
   An investment bank models the expected performance of its assets over a five- day period. Over that period, the return on the bank's portfolio, i, has a mean value of 0.1% and standard deviation 0.2%. $(1 + i)$ is lognormally distributed. Calculate the value of j such that the probability that i is less than or equal to j is 0.05.

6. Sep 01 - Q6
   $(1 + i_t)$ follows a log normal distribution where i_t is the rate of interest over a given time period beginning at time t. The parameters of the distribution are $\mu = 0.06$ and $\sigma^2 = 0.0009$.

   Calculate the inter-quartile range for the accumulation of 100 units of money over the given time period, beginning at time t.

7. Sep 02 - Q8
   £10,000 is invested in a bank account which pays interest at the end of each year.

   The rate of interest is fixed randomly at the beginning of each year and remains unchanged until the beginning of the next year. The rate of interest applicable in any one year is independent of the rate applicable in any other year.

   During the first year the rate of interest per annum effective will be one of 3%, 4% or 6% with equal probability.

   During the second year, the rate of interest per annum effective will be either 5% with probability 0.7 or 4% with probability 0.3.

  (a) Assuming that interest is always reinvested in the account, calculate the expected accumulated amount in the bank account at the end of two years.

  (b) Calculate the variance of the accumulated amount in the bank account at the end of two years.

8. Apr 97(Subject A1) - Q3
   Let i_t denote the effective rate of interest in the year t to $t + 1$. It is